About the Mathematics: Informal Notation – Arrow Notation

Alternative informal representation of student thinking
Arrow Notation to record Addition & Subtraction Strategies

Using *Arrow Notation* to Capture Students Thinking

Leads to arrow (→) – The arrow is gaining traction in the mathematics education community as an acceptable symbol to capture the action by the individual as the computation unfolds. The symbol is referred to as a "*leads to"* arrow or "*gets me to…"* arrow. The benefits for introducing arrow notation to students is two-fold: it avoids abuse of the equal sign and it potentially captures movement on the number line

Avoiding abuse of the equal sign – Arrow notation is an informal representation used to capture students' thinking as the increment forward or backward in the numbers. This notation prevents students from abusing the equal sign in capturing the steps in their thinking. While my thinking may be to split 5 into 3 and 2 so that I can add 3 to 7 and then add 2 to get to 12, *it is incorrect to write* that as…

$$7 + 3 = 10 + 2 = 12$$

7 + 3 does equal 10, but 7 + 3 ≠ 10 + 2. Students need to view the equal sign as a *comparison symbol* of two equal mathematical expressions, not as an operation symbol. Thus the above example can be captured as…

$7 + \underline{3} \rightarrow 10 + \underline{2} \rightarrow 12$; 12 is my answer

Mimicking movement on a number line – *Arrow Notation* can be used to capture the direction of movement on the number line depending on how it is used, particularly in subtraction. Consider the two ways it is used below to capture a solution to 42 + *c* = 100.

$$42 + 50 \longrightarrow 92 + 8 \longrightarrow 100$$

$$42 \xrightarrow{\qquad\quad +50 \qquad\quad} 92 \xrightarrow{+8} 100$$

$c = 58$

In the first instance, the arrows are capturing a line of thinking. In the second, a relative value is being reflected as to the length of distance each span represents. While it is not an exact representation nor the proportions of a true number line, it reflects an individual's understanding of both the direction of movement and relative span of distance. The same thing holds for the use of arrow notation to capture one's thinking in solving a subtraction problem.

$$72 - 48$$
$$72 - 40 \longrightarrow 32 - 2 \longrightarrow 30 - 6 \longrightarrow 24$$

$$24 \xleftarrow{-6} 30 \xleftarrow{-2} 32 \xleftarrow{\qquad\quad -40 \qquad\quad} 72$$

In the first solution, the arrow notation is being used to capture the continuous flow of removing increments of 48; first the 40, then 2 of the eight and then the final 6 of the eight. In the second, the use of the arrow notation captures how the solution moves along a number line. Both representations are accurate. However, each reflects a different mathematical idea. If your role in an instructional context with students is to be the recorder of a student's thinking, you need to decide and record which representation is most mathematically productive to the student(s) to consider at that time.

Range of strategies - When posing the *Get to a Number* activity with students, know that there is more than one strategy a student can use to find a solution. It is essential that students understand that flexibility in thinking is acceptable and admired. What may involve you is the support of students in how to record those various strategies. Once recorded, a conversation that compares and contrasts the different strategies focuses students on the various possibilities available to them in the future. Compare the following strategies to solve the following. *You are at 678, how much to get to 1000.*

678 —(+20)→ 698 —(+2)→ 700 —(+300)→ 1000; The answer is 322

678 —(+300)→ 978 —(+2)→ 980 —(+20)→ 1000; The answer is 322

678 —(+400)→ 1000 → 1078
←(-78)— 400 – 78 = 322

Each of these strategies is efficient and productive. The first strategy uses smaller increments to get to the next 100, the second increments a large quantity first and then smaller amounts to 1000, the third, and example of *the compensation strategy*, goes beyond the target amount of 1000 and then adjusts at the end to find the final solution.

Examples for subtraction for 23.14 – 19.89

The Difference Between Strategy (subtraction)
19.89 ←(-.11)— 20 ←(- 3)— 23 ←(-.14)— 23.14; The answer is 3 + .11 + .14 = 3.25

Separate Result Unknown
.89 = .14 + .75
3.25 ←(-.75)— 4 ←(-19)— 23 ←(-.14)— 23.14 The answer is 3.25

Compensation Strategy
3.14 ←(- 20)— 23.14
—(+.11)→ *3.25* The answer is 3.25

The difference between (addition)
19.89 —(+.11)→ 20 —(+3.14)→ 23.14 The answer is 3.14 + .11 = 3.25

The intent is to honor the students' thinking and to support them in representing their strategy as mathematically accurate as possible.

Standard Representation of Mathematical Equations - Eventually, students can move to more conventional mathematical forms of representation. This does not mean that vertical/column forms of representation are more powerful than horizontal representations. Remember, upper-level mathematics is all represented in a horizontal format. It is in accounting and spreadsheets where vertical formats predominate. Students *should be comfortable moving across both formats* of representation. The mathematical relationships show up differently in the two formats and it is powerful for students to understand how the same mathematical idea is captured in both representational forms.