SMP8: Look for and express regularity in repeated reasoning

While Minnesota has not adopted the Common Core State Standards for Mathematics, there are many aspects of the Common Core—particularly its Standards for Mathematical Practice (SMP)—that are helpful for all learners. In this post, I'll address SMP8: *Look for and express regularity in repeated reasoning.* This mathematical practice is described in the Common Core as follows: "Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts." But how might such *repeated calculations* look in the classroom?

Below are some problems, adapted from an NSF-funded curriculum, the *CME Project*.[1]

1. Suppose line k contains the points $A(5, -3)$ and $B(11, 1)$.
   (a) What is the slope between the points given?
   (b) Explain why the following proves that the point $(14, 3)$ is on line k:
   $$\frac{3-1}{14-11} = \frac{2}{3}.$$
   (c) Explain why the point $(17, 5)$ is on line k.
   (d) Is $(8, -1)$ on line k? How about $(32, 15)$? $(12, 2)$? $(-4, -9)$?
2. (a) Given a point (x, y), how can you tell whether or not it's on line k? Explain.
   (b) Write an equation for k.
3. Use the method from Problem #1 to find an equation for line k. But this time, use point A as the "base point." Do you get the same equation? Explain.

The first problem prompts the students to repeatedly check, through slope calculation, whether or not a point is on line k. These repeated calculations provide students with a feel for the concept at hand and help them to work towards the formula $(y-1)/(x-11) = 2/3$. The second problem guides the students through the generalization process—students should see that it's just a matter of replacing concrete points like $(17, 5)$ and $(12, 2)$ with a more general (x, y). In the third problem, students explore how two equations that seem different are actually equivalent.

Fostering mathematical practices such as SMP8—the essence of what it means to *do* mathematics—in students is just as important (if not more important) than teaching them a specific set of facts. I end with a quote from a paper by Cuoco, Goldenberg, and Mark:[2]

> Much more important than specific mathematical results are the habits of mind used by the people who create those results. ... Although it is necessary to infuse courses and curricula with modern content, what is even more important is to give students the tools they will need in order to use, understand, and even make mathematics that does not yet exist.

[1] Education Development Center, Inc. (2009). *CME Project: Algebra 1.* Boston, MA: Pearson.

[2] Cuoco, A., Goldenberg, E. P., & Mark, J. (1996). Habits of mind: An organizing principle for mathematics curriculum. *Journal of Mathematical Behavior, 15*(4), 375–402.